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QCD analysis of the CCFR data for xF_3 and Higher–Twist Contribution.

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Abstract

The QCD analysis of the xF_3 structure function measured in deep-inelastic scattering of neutrinos and antineutrinos on an iron target at the Fermilab Tevatron is done in 1–, 2– and 3–loop order of QCD. The x dependence of the higher–twist contribution is evaluated. The experimental value of higher–twist corrections to the Gross–Llewellyn Smith sum rule is discussed.

At present, the precise measurements of structure functions (SF) and detailed theoretical calculations of QCD predictions for scaling violations (up to 3–loop order for $xF_3(x,Q^2)$ and $F_2(x,Q^2)$) provide an important means of accurate comparison of QCD with experiment. The importance of higher–twist (HT) contribution to SF was pointed from the very beginning of QCD comparison with experimental data [1] on SF. Despite a fast progress in theoretical QCD calculations of power corrections to nonsinglet SF and sum rules [2, 3] (for reviews and references see [4]), the shape of HT (order $1/Q^2$) contributions is measured only for F_2 SF [5] and is still only estimated for xF_3 [6]. In the present note, the x dependence of HT contribution is phenomenologycally determined in the framework of QCD analysis of the experimental data of the CCFR collaboration [1] obtained at Fermilab Tevatron [7] for the xF_3 structure functions of the deep-inelastic scattering of neutrinos and antineutrinos on an Iron target by means of the Jacobi polynomial expansion method in the 1–, 2– and 3–loop order of QCD.

The details of this method are described in [8]-[13]. The Q^2 - evolution of the moments $M_3^{QCD}(N,Q^2)$ is given by perturbative QCD [14, 15].

$$\begin{aligned} M_3^{QCD}(N,Q^2) &= \left[\frac{\alpha_S\left(Q_0^2\right)}{\alpha_S\left(Q^2\right)}\right]^{d_N} H_N\left(Q_0^2,Q^2\right) M_3^{QCD}(N,Q_0^2), \quad N=2,3,... \qquad (1)\\ d_N &= \gamma^{(0),N}/2\beta_0,. \end{aligned}$$

Here $\alpha_s(Q^2)$ is the constant of strong interaction, $\gamma_N^{(0)NS}$ are nonsinglet leading order anomalous dimensions. The factor $H_N\left(Q_0^2,Q^2\right)$ contains all next– and next–to–next–to–leading order QCD corrections and is constructed in accordance with [13] based on theoretical results of [16].

Having at hand the moments (1) and following the method [9, 10], we can write the structure function xF_3 in the form:

$$xF_3^{pQCD}(x,Q^2) = x^{\alpha}(1-x)^{\beta}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\beta)M_3^{QCD}\left(j+2,Q^2\right), \qquad (2)$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha,\beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers of x:

$$\Theta_n^{\beta}(x) = \sum_{j=0}^{n}c_j^{(n)}(\beta)x^j. \qquad (3)$$

The unknown coefficients $M_3(N,Q_0^2)$ in (1) could be parametrised as Mellin moments of some function:

$$M_3^{QCD}(N,Q_0^2) = \int_0^1 dx x^{N-2}Ax^b(1-x)^c(1+\gamma x), \quad N=2,3,... \qquad (4)$$

For $N_{max}=8$ the accuracy better than 10^{-3} is achieved in a wide region of parameters α and β [9]. In particular, we use $\alpha=0.7$ and $\beta=3.0$

[1]Announced by CCFR collaboration reevaluation of the structure functions could change the results of the QCD analysis.

Using Mellin moments (1),(4), expression (2) for SF and taking target–mass corrections (TMC) into account, we have reconstructed $xF_3^{pQCD}(x,Q^2)$. Five free parameters: A, b, c, γ and QCD parameter $\Lambda_{\overline{MS}}$ are to be determine from comparison with experimental data.

To extract the HT, contribution we parameterize the nonsinglet SF as follows:

$$xF_3(x,Q^2) = xF_3^{pQCD}(x,Q^2) + h(x)/Q^2, \qquad (5)$$

where the Q^2 dependence of the first term in the r.h.s is determined by perturbative QCD. Constants $h(x_i)$ (one per x–bin) parameterize the HT x dependence. In accordance with the x-bin structure of the CCFR data we put $x_i = 0.015,\ 0.045,\ 0.080,$ $0.125,\ 0.175,\ 0.225,\ 0.275,\ 0.350,\ 0.450,\ 0.550,\ 0.650$ for $i = 1, 2...11$. The values of constants $h(x_i)$ as well as parameters A, b, c,γ and scale parameter Λ are determined by fitting the set of the CCFR data at 90 experimental points of xF_3 in a wide kinematical region: $1.3\ GeV^2 \le Q^2 \le 501\ GeV^2$ and $0.015 \le x \le 0.65$ and $Q_0^2 = 10\ GeV^2$. We have put the number of flavours to equal 4. The TMC are taken into account to the order of $o(M_{nucl}^4/Q^4)$. The nuclear effect of the relativistic Fermi motion is estimated ¿from below by the ratio $R_F^{D/N} = F_3^D/F_3^N$ [18] obtained in the covariant approach in light-cone variables [17].

Results of the fit are presented in Table 1 and Figures 1-3. The theoretical prediction for $h(x)$ from [3] is presented at Figure 3.

Several comments are in order:

- A decrease of $\chi^{2(NNLO)}$ in comparison with $\chi^{2(NLO)}$ and $\chi^{2(LO)}$: $\chi^{2,NNLO}_{d.f.} < \chi^{2,NLO}_{d.f.} < \chi^{2,LO}_{d.f.}$ demonstrates that 3–loop effects are important for the kinematical region under consideration. For all orders of QCD the χ^2 per degree of freedom is smaller than in [13], where the fit was done without HT contribution.

- The obtained value of the Λ is smaller in comparison with results of the previous analysis of CCFR data [12, 13] with the cut off $Q^2 > 10\ GeV^2$ $\Lambda_{\overline{MS}}^{NNL)} = 184 \pm 31\ MeV$ but exhibits relatively large statistical errors. Results of the NNLO fit gives the constant of strong interaction $\alpha_S^{NNLO}(M_Z^2) = 0.104^{+0.006}_{-0.008}(syst.)$ in agreement within the errors with usual DIS results [19] and with the predictions of CCFR-NuTeV Collaboration [20] based on the test of the Gross–Llewellyn Smith (GLS) sum rule.

- The shape of h(x) demonstrates for LO, NLO and NNLO fit a very small value at $0.015 \le x \le 0.045$, a negative value at $0.1 \le x \le 0.045$ (with a minimum located at about $x = 0.2$) and increase from a negative to a positive value at $0.2 \le x \le 0.65$. This behavior is in qualitative agreement with theoretical predictions of [3] and reproduces appropriately the predicted zero of h(x): $x^{theor} \sim 0.67$ while in our NNLO analysis $x^{NNL} \sim 0.40$. A separate fit with cuts off $Q^2 > 5\ GeV^2$ and $Q^2 > 10\ GeV^2$ shows the stability of shape of h(x) and increase of errors.

- The absolute value of h(x) slightly decreases from LO to NNLO fit. It may be indicates a special role of higher order perturbative QCD corrections reveals by renormalon technique [25]: at higher order xF_3^{pQCD} in (5) describes effectively the power corrections.

- Definite theoretical predictions are presented for the first moment of h(x) which contributes to the GLS sum rule [21]: $h_1 = \int_0^1 \frac{h(x)}{x}dx$. A general structure of this contribution is known ¿from the results of Ref.[22] The corresponding numerical calculations of this term was made in Ref. [23] $h_1 = -0.29 \pm 0.14$[2] and more recently in Ref. [24] $h_1 = -0.47 \pm 0.04$, using the same three-point function QCD–sum–rules technique. One can estimate h_1 based on the results of Table 1. : $h_1^{LO} = 0.12 \pm 0.53$, $h_1^{NLO} = 0.14 \pm 0.53$ and $h_1^{NNLO} = 0.13 \pm 0.45$. Taking into account the errors the values of h_1^{LO}, h_1^{NLO} and h_1^{NNLO} could be compared with the prediction of [23] and the recent result of [25] for GLS sum rule:

$$\text{GLS} \;=\; 3\left\{\left[1 - \frac{\alpha_s(Q)}{\pi} + \ldots \pm \frac{0.02 - 0.07}{Q^2}\right] - \frac{(0.1 \pm 0.03)}{Q^2}\right\} + O(1/Q^4)$$

  It should be noted that the fit without the nuclear effect $R_F^{D/N} = 1$ provides $h_1^{LO,R=1} = 0.11 \pm 0.51$, $h_1^{NLO,R=1} = 0.12 \pm 0.40$ and $h_1^{NNLO,R=1} = 0.12 \pm 0.48$ in a good agreement with previous results. The large contribution of small x region to h_1 needs the shadowing correction taking into account for more detail analysis [26].

In conclusion it should be stressed, that for precise determination of the HT contribution to SF the role of nuclear effect should be clarified and a more realistic approximation for $R_F^{Fe/N} = F_3^{Fe}/F_3^N$ is needed. A possible interplay of the nuclear effect and TMC was considered in [27]. We also did not take into account the threshold effects on Q^2 evolution of SF due to heavy quarks [28] which is necessary owing to a wide kinematical region of data under consideration.

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Figure captions.

Fig.1. Higher–twist contributions from LO fit and the theoretical prediction for $h(x)$ from [3].

Fig.2. Higher–twist contributions from NLO fit.

Fig.3. Higher–twist contributions from NNLO fit.

Table I. Results of 1-, 2- and 3- order QCD fit (with TMC) of the CCFR xF_3 SF data for $f = 4$, $Q^2 > 1.3GeV^2$ with the corresponding statistical errors and values of $h(x)$ at different values of x. $N_{MAX} = 10$ for 1- and 2- oder and $N_{MAX} = 7$ for 3- order fit.

Table I.

	LO	NLO	NNLO
$\chi^2_{d.f.}$	65.1/74	62.9/74	60.9/74
A	6.69 ± 0.87	6.04 ± 0.51	5.56 ± 0.18
b	0.772 ± 0.040	0.745 ± 0.026	0.719 ± 0.011
c	4.04 ± 0.16	3.97 ± 0.14	3.91 ± 0.12
γ	0.424 ± 0.53	0.603 ± 0.317	0.707 ± 0.055
$\Lambda_{\overline{MS}}$ $[MeV]$	76 ± 62	132 ± 80	134 ± 57
x_i	$h(x_i)$ $[GeV^2]$		
0.015	0.012 ± 0.034	0.018 ± 0.036	-0.015 ± 0.022
0.045	-0.008 ± 0.049	0.037 ± 0.063	0.043 ± 0.054
0.080	-0.199 ± 0.061	-0.107 ± 0.079	-0.067 ± 0.077
0.125	-0.318 ± 0.084	-0.203 ± 0.083	-0.144 ± 0.086
0.175	-0.175 ± 0.133	-0.073 ± 0.114	-0.005 ± 0.106
0.225	-0.242 ± 0.186	-0.176 ± 0.159	-0.113 ± 0.133
0.275	-0.217 ± 0.241	-0.202 ± 0.210	-0.162 ± 0.168
0.350	0.095 ± 0.294	0.023 ± 0.253	0.011 ± 0.185
0.450	0.129 ± 0.302	-0.010 ± 0.280	-0.051 ± 0.207
0.550	0.283 ± 0.235	0.150 ± 0.249	0.086 ± 0.205
0.650	0.510 ± 0.155	0.412 ± 0.180	0.349 ± 0.159

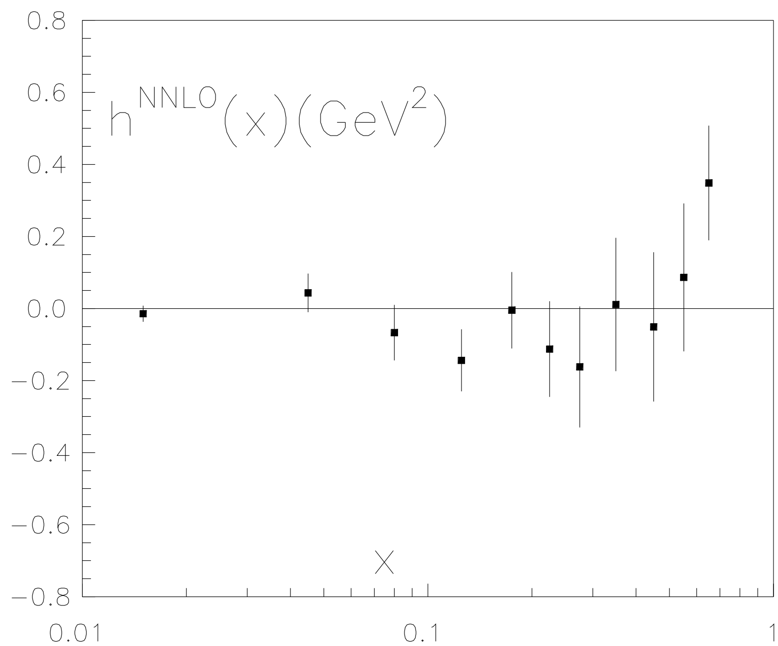

Fig. 3.

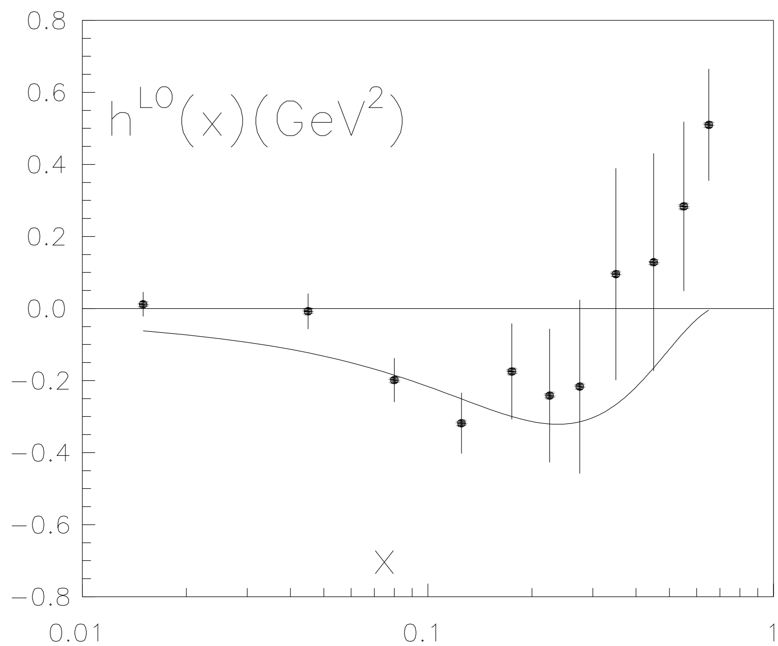

Fig. 1.

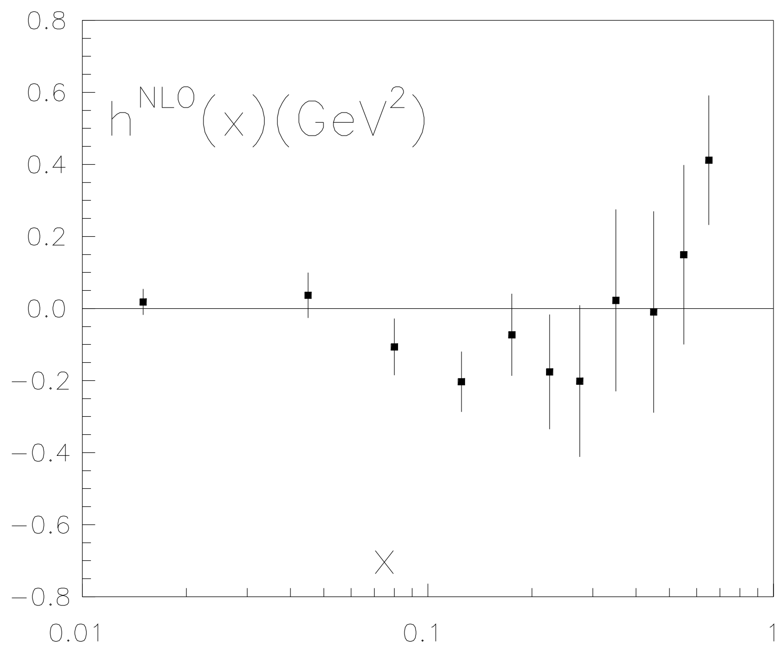

Fig. 2.